

MR2957009 (Review) 01A20**Ambrosi, Gerhard Michael (D-TRR-4)****Pre-Euclidean geometry and Aeginetan coin design: some further remarks. (English summary)***Arch. Hist. Exact Sci.* **66** (2012), no. 5, 557–583.1432-0657

In 1990, Benno Artmann [Math. Intelligencer **12** (1990), no. 4, 43–50; [MR1076534 \(91i:01006\)](#)] observed in a paper concerned with geometric motifs on ancient Greek coins that those from Aegina postdating 404 BCE (when the population was allowed to return from exile) carried a motif very close to the diagram of *Elements* II.4 on the reverse, whereas those antedating 431 (when Athens expelled the population) display other patterns—in particular, from c. 480 onward, a square divided by skew lines in five parts. The obverse always shows a very precisely executed tortoise.

In 2008, David Aboav suggested [Arch. Hist. Exact Sci. **62** (2008), no. 6, 603–612; [MR2457063 \(2009h:01004\)](#)] that this skew diagram was meant to divide the square in equal parts. For this purpose, he silently replaced Artmann's illustration where this is obviously neither the case nor the intention by another one (for which he gives no reference, not even mentioning that a substitution has taken place) where equality might have been intended. The point of his article is that this division can be performed exactly by means of the last proposition of (Raymond Archibald's reconstruction of) Euclid's *Book on Divisions*.

Given the high quality of the tortoises, it is already improbable that the obvious deviation of Artmann's specimen from equality should be the outcome of sloppy work. Worse, if one investigates the whole development (as can be done adequately on the British Museum Catalogue of coins from Attica, Megaris and Aegina [B. V. Head, *Catalogue of Greek coins. Attica—Megaris—Aegina*, Longmans, London, 1888]; see archive.org/details/acataloguegreek00medagoog) one will discover that the skew pattern develops over time from a square divided into eight equal triangles by the diagonals and parallel transversals; at some moment, five (sometimes only four, sometimes six) of these become incused while three remain filled. These triangles then begin to be placed more or less symmetrically, in the end filling the whole square and in this way producing the skew pattern. Nothing in this development suggests an intention of equal areas, and many specimens contradict it. Quite apart from its dubious speculations about Pythagorean influence, Aboav's paper is hence better disregarded.

Nonetheless, Aboav's paper is the starting point for the present author, who takes the intention of equal areas for granted (while refuting Aboav's general conclusions) and suggests a post-404 transformation of this pattern into the one recalling *Elements* II.4. He adds a claim that the ratio between the areas the two subsquares of this diagram is $2 : 1$, and that between their sides is thus $\sqrt{2} : 1$ (leaving out a few specimens where the latter ratio is clearly much larger, namely around $1.8 : 1$). He then shows how such a diagram can be constructed by means of a quartered circle, where a square is inscribed in one quarter and another one circumscribed on the quarter opposing it diagonally. This he connects to Plato's *Meno* and *Timaeus*, to the golden section, to Hippocrates'

lunules, etc.

Unfortunately, even here the empirical basis contradicts the starting point. Of 17 specimens which the reviewer has measured (those from the British Museum, and web reproductions from Altes Museum in Berlin and from various British collections), three exhibit a ratio of the sides between 1.8 : 1 and 1.85 : 1 (always measuring from the outer edge of the incision to the middle of the separating ridge). The remaining 11 specimens have a mean value of 1.37, a standard deviation of 0.15, and no tendency at all to cluster around 1.4. There is a kind of cluster between 1.18 and 1.4, with mean value 1.28 and standard deviation 0.08, and then five specimens falling between 1.46 and 1.64. Once again, given the care with which the turtles are depicted, any attempt to approach the ratio $\sqrt{2} : 1$ can be safely discarded.

On two specimens, the letter A or the letter I is inscribed in the small square. The author takes them to stand for the number 1 in the alphabetic and the Attic numeral system, respectively. He admits (footnote 7) that one specimen which he knows about has A in the square and continues IΓ in the adjacent rectangle, but that does not disturb him. He is unaware that others (with a left-right inversion of the diagram) have A IΓ in the rectangle continuing with I in the square, or still other letters (Δ, E, X etc.)—as pointed out in the British Museum catalogue, these, when not abbreviating A IΓ I N A, stand for the names of magistrates in office.

Beyond that, the author takes the circles, etc., which he superimposes on his reconstructed diagrams as the “explicit topic of the coin design” (p. 581).

Since the empirical basis is absent, there is no reason to investigate (nor summarize) the derived hypotheses. In the author’s opinion, “since there is no documentary evidence about [the Aeginetan coins], we must rely on speculation” (p. 580), and “the coins’ significance lies not just in the thoughts which went into their design but also in the ones which they provoked [in the mind of the modern author]” (p. 582). The question is whether such private impressions of “Greek mathematical ingenuity” (p. 582) were not better published on a private web site. So far, the late Benno Artmann has the last reliable word on the coins from Aegina.

Reviewed by *Jens Høyrup*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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